# Lecture No. 15

# Colliders and Luminosity

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# Introduction

**Colliders for high energy particle physics experiments are surely one of the most important application of particle accelerators.**

**Actually, the developments in particle accelerators and of elementary particle physics probably represent one of the best examples of synergy between different physics disciplines.**

**Colliders can be characterized by the different nature of the colliding particles (leptons or hadrons) and by the different acceleration scheme used (linear or circular)**

**In existing lepton colliders, electrons collide with positrons and a significant R&D is undergoing for the definition of a possible scheme for a muon collider.**

**Hadron colliders include, protons colliding with protons or anti-protons and heavy ion colliders.**

**Higher collision energies can be achieved with hadron colliders but "cleaner" measurements can be done with lepton colliders.**

# Introduction

In electron-positron collisions the particles annihilate and all the energy at the center of mass system is available for the generation of elementary particles.

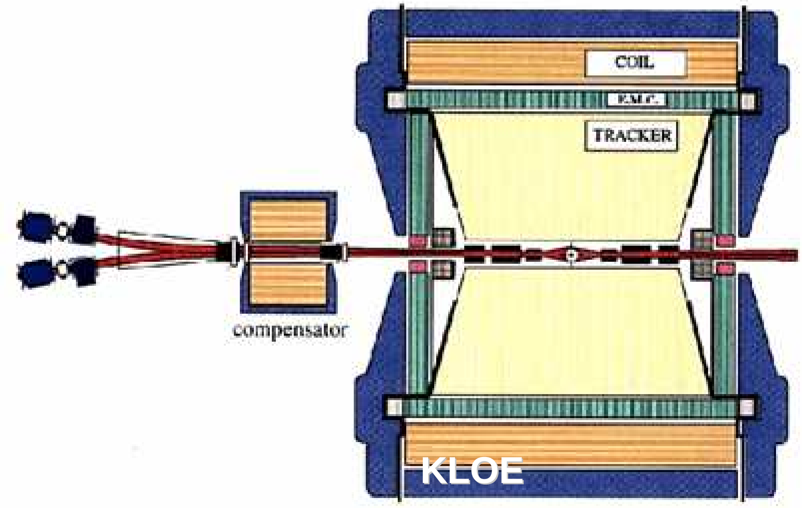
Such particle generation can happen only if it exists a particle with rest mass energy equal to the collision energy at the center of mass system.

The energy of the colliding beams can be tuned on the rest mass energy of a known particle for studying its properties, or can be scanned for the research of unknown particles.

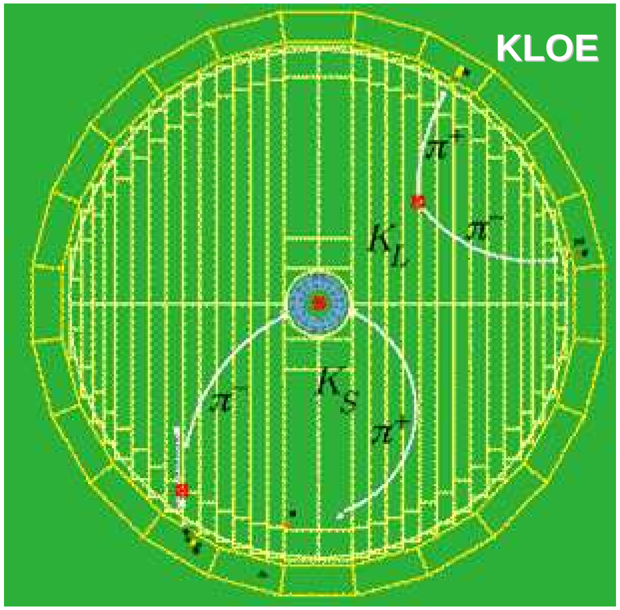
In hadron colliders, the quarks in the hadrons interact during the collision and generate other particles. Because each hadron is a combination of three quarks, simultanoeus generation of different particles is possible.

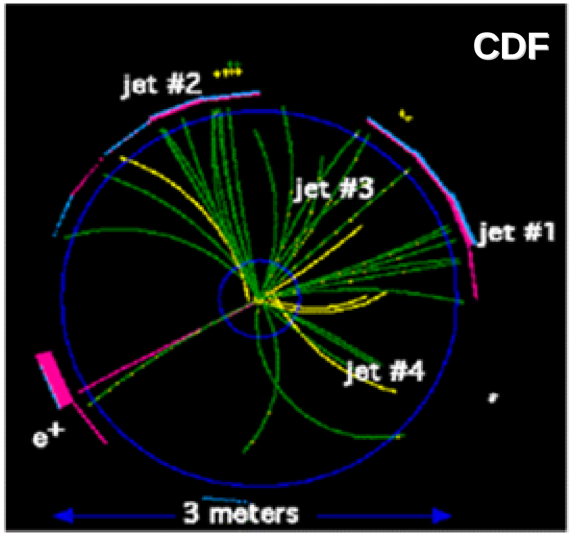
Most of the particles generated during a collisions usually have a short lifetime and decay in other particles. Particles detectors are designed in order to measure the particle itself when possible or to measure the particles generated during the decay.

# High Energy Physics Detectors







# Energy at the Center of Mass System

- Two particles have equal rest mass $m_0$.

  **Center of Mass Frame (CMF):** Velocities are equal and opposite, total energy is $E_{cm}$.

$$P_1 = \left(E_{CM}/2c, p\right) \qquad P_2 = \left(E_{CM}/2c, -p\right)$$

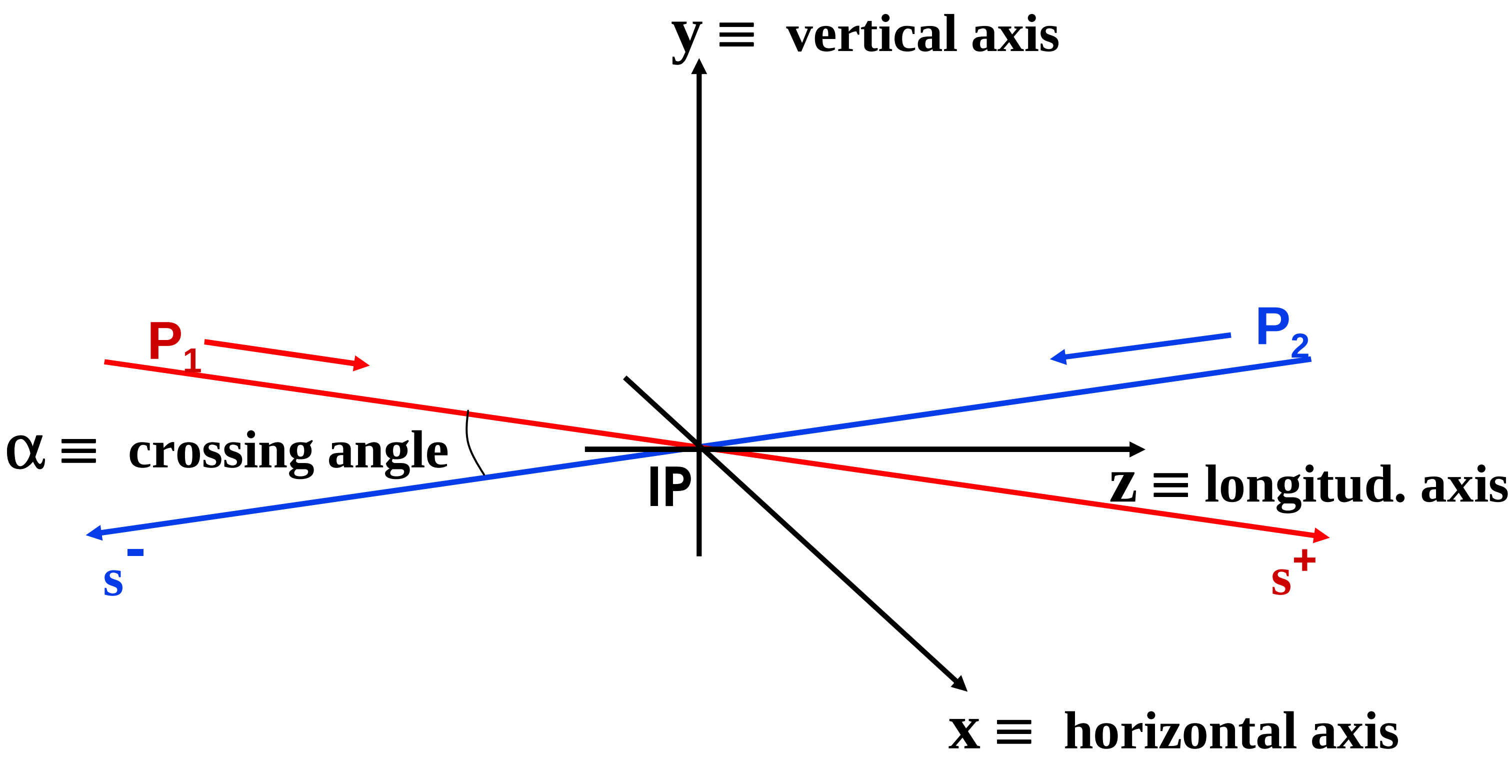
**Laboratory frame (LF):**

$$\tilde{P}_1 = \left(E_1/2c, p_1\right) \qquad \tilde{P}_2 = \left(E_2/2c, p_2\right)$$

- The quantity $\left(P_1 + P_2\right)^2$ is invariant.
- In the CMF, we have $\left(P_1 + P_2\right)^2 = E_{CM}^2/c^2$
- While in the LF: $\left(\tilde{P}_1 + \tilde{P}_2\right)^2 = \tilde{P}_1^2 + \tilde{P}_1^2 + 2\tilde{P}_1\tilde{P}_2 = 2m_0^2c^2 + 2\tilde{P}_1\tilde{P}_2$
- And after some algebra we can obtain for relativistic particles:

$$\boxed{E_{cm} \cong 2\sqrt{E_1 E_2}}$$

# Interaction Region Reference Frame




$\{ x, y, z \} \equiv$ Lab. Reference Frame

# Basic Definitions

**Cross Section:**

Event Rate
per Unit Incident Flux
per Target Particle

**Luminosity:**

Event Rate
for a
Unit Cross Section Event

# Counter Rotating Beams Case

$$n_\pm(x,y,z,t)$$

$$\iiint dx\,dy\,dz\; n_\pm(x,y,z,t) = N_\pm$$

- Single Bunch
- Head-on Collision
- Counter-rotating Beams with Longitudinal Speed $v$
- Revolution Frequency $f_R$

$$L = 2v\, f_R \iiiint dx\,dy\,dz\,dt\; n_+(x,y,z,t)\, n_-(x,y,z,t)$$

# Gaussian Beam
# Single Bunch Luminosity

$$n_-(x,y,z,t) = N_- \frac{e^{-\frac{x^2}{2\sigma_{x-}^2} - \frac{y^2}{2\sigma_{y-}^2} - \frac{(z-vt)^2}{2\sigma_{z-}^2}}}{(2\pi)^{3/2}\,\sigma_{x-}\sigma_{y-}\sigma_{z-}} \qquad n_+(x,y,z,t) = N_+ \frac{e^{-\frac{x^2}{2\sigma_{x+}^2} - \frac{y^2}{2\sigma_{y+}^2} - \frac{(z+vt)^2}{2\sigma_{z+}^2}}}{(2\pi)^{3/2}\,\sigma_{x+}\sigma_{y+}\sigma_{z+}}$$

$$\sigma_{x\pm}, \sigma_{y\pm} \equiv \text{constants}$$

$$L = f_R \frac{N_+ N_-}{2\pi\sqrt{\left(\sigma_{x+}^2 + \sigma_{x-}^2\right)\left(\sigma_{y+}^2 + \sigma_{y-}^2\right)}}$$

$$L = f_R \frac{N_+ N_-}{4\pi\ \sigma_x \sigma_y}$$

$$\sigma_{x+} = \sigma_{x-} \qquad \sigma_{y+} = \sigma_{y-}$$

# Two Main Effects

**Geometric Effects:**

How the "geometry" of the interaction point (IP) and the size of the beams affect luminosity

**Charge Related Effects:**

Or *beam-beam* effects. Charge plays a major role, limiting the achievable luminosity in most of storage ring colliders.

# Geometric Luminosity: Collider Parameters

$$L = f_R \frac{N_+ N_-}{4\pi\, \sigma_x^* \sigma_y^*}$$

$$\eta_x = \eta_y = 0$$

$$\sigma_y^* = \sqrt{\frac{\kappa}{1+\kappa}\varepsilon \beta_y^*} \qquad \sigma_x^* = \sqrt{\frac{1}{1+\kappa}\varepsilon \beta_x^*}$$

$$\boxed{L = f_R \frac{N_+ N_- (1+\kappa)}{4\pi\, \varepsilon \sqrt{\kappa \beta_x^* \beta_y^*}} \cong f_R \frac{N_+ N_-}{4\pi\, \varepsilon \sqrt{\kappa \beta_x^* \beta_y^*}}}$$

# Geometric Luminosity

- Very low currents

↓

- Negligible beam-beam effects

$$L = 2 f_R \frac{v}{c} \cos\alpha \sqrt{c^2 - v^2 \sin^2\alpha} \iiiint dxdydzdt \quad f_+ f_-$$

- Crossing angle $\alpha$

$$f_+ = \frac{N_+ e^{-\frac{(x\cos\alpha - z\sin\alpha)^2}{2\sigma_{x+}^2(x,z)} - \frac{y^2}{2\sigma_{y+}^2(x,z)} - \frac{(x\sin\alpha + z\cos\alpha - vt)^2}{2\sigma_{z+}^2}}}{(2\pi)^{3/2} \ \sigma_{x+}(x,z) \ \sigma_{y+}(x,z) \ \sigma_{z+}}$$

- Horizontal & vertical Offset $\Delta x$ and $\Delta y$

$$\sigma_{k+}^2 = \sigma_{k+}^{*2}\left[1 + \frac{(x\sin\alpha + z\cos\alpha - \Delta w_+)}{\beta_{k+}^{*2}}\right] \qquad k = x, y$$

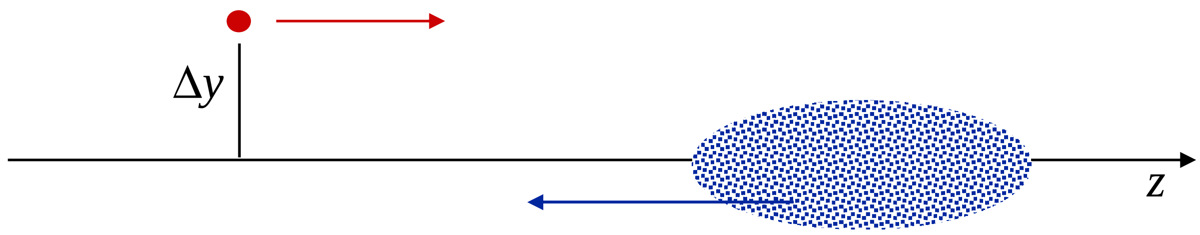
- IR position $\Delta t$

$$f_- = \frac{N_- e^{-\frac{[(x-\Delta x)\cos\alpha + z\sin\alpha]^2}{2\sigma_{x-}^2(x,z)} - \frac{(y-\Delta y)^2}{2\sigma_{y-}^2(x,z)} - \frac{[-(x-\Delta x)\sin\alpha + z\cos\alpha + v(t-\Delta t)]^2}{2\sigma_{z-}^2}}}{(2\pi)^{3/2} \ \sigma_{x-}(x,z) \ \sigma_{y-}(x,z) \ \sigma_{z-}}$$

- Different beta star for the two beams

$$\sigma_{k-}^2 = \sigma_{k-}^{*2}\left[1 + \frac{(-x\sin\alpha + z\cos\alpha - \Delta w_-)}{\beta_{k-}^{*2}}\right] \qquad k = x, y$$

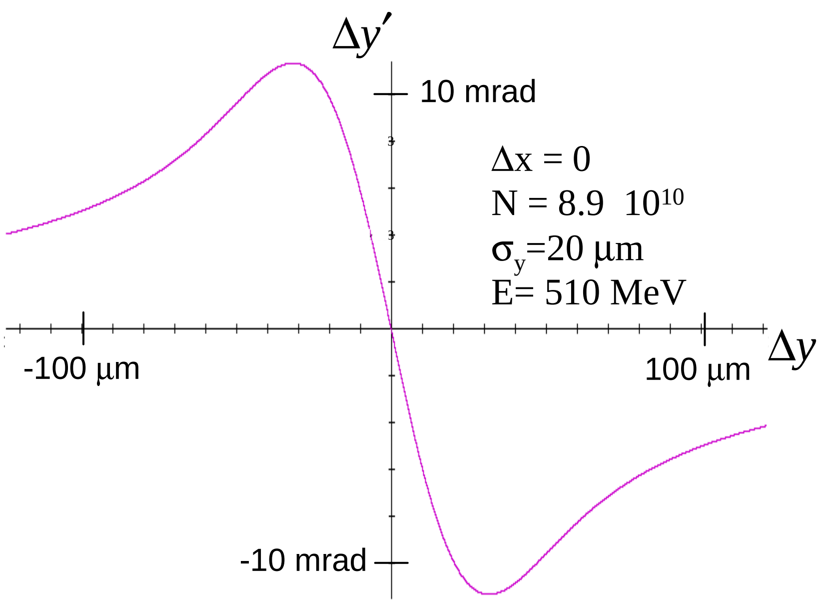
# Beam-Beam Effects




## For a gaussian charge distribution:

$$\Delta y' = -\frac{2 N r_e \Delta y}{\gamma} \int_0^\infty \frac{\exp\left(-\frac{\Delta x^2}{2\sigma_x^2 + w} - \frac{\Delta y^2}{2\sigma_y^2 + w}\right)}{\left(2\sigma_y^2 + w\right)^{\frac{3}{2}}\left(2\sigma_x^2 + w\right)^{\frac{1}{2}}} dw$$

$$\Delta x' = -\frac{2 N r_e \Delta x}{\gamma} \int_0^\infty \frac{\exp\left(-\frac{\Delta x^2}{2\sigma_x^2 + w} - \frac{\Delta y^2}{2\sigma_y^2 + w}\right)}{\left(2\sigma_y^2 + w\right)^{\frac{1}{2}}\left(2\sigma_x^2 + w\right)^{\frac{3}{2}}} dw$$



# Linear Beam-Beam

$$\Delta y \ll \sigma_y^* \qquad \Delta x \ll \sigma_x^*$$

$$\Delta y' \cong -\frac{1}{f_y}\Delta y \qquad f_y^{-1} = \frac{2 N r_e}{\gamma \sigma_y^* \left(\sigma_x^* + \sigma_y^*\right)}$$

$$\Delta x' \cong -\frac{1}{f_x}\Delta x \qquad f_x^{-1} = \frac{2 N r_e}{\gamma \sigma_x^* \left(\sigma_x^* + \sigma_y^*\right)}$$

**Focusing Quadrupole (thin lens):**

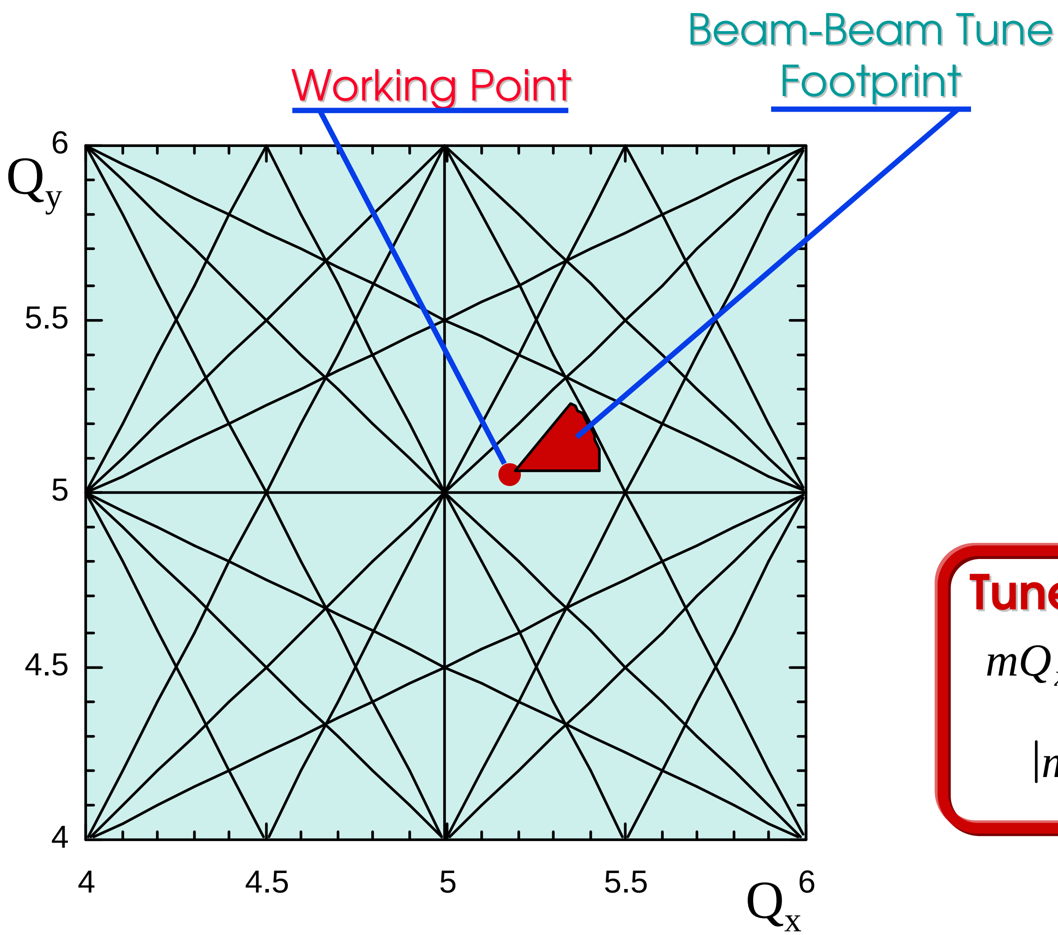
$$\mathbf{Q}_F^k \equiv \begin{pmatrix} 1 & 0 \\ -1/f_k & 1 \end{pmatrix} \qquad k = x, y$$

**Beam-Beam Deflection (off-center particles)**

**Linear Beam-Beam Tune Shift**

$$\xi_y^+ = \frac{N_- r_e \beta_y^{*+}}{2\pi\gamma\, \sigma_y^- \left(\sigma_y^- + \sigma_x^-\right)} = \Delta Q_y \qquad \xi_x^+ = \frac{N_- r_e \beta_x^{*+}}{2\pi\gamma\, \sigma_x^- \left(\sigma_y^- + \sigma_x^-\right)} = \Delta Q_x$$

# Choice of the Working Point




**Tune Resonances**

$$mQ_x + nQ_y = p \qquad m, n, p \in \mathbf{N}$$

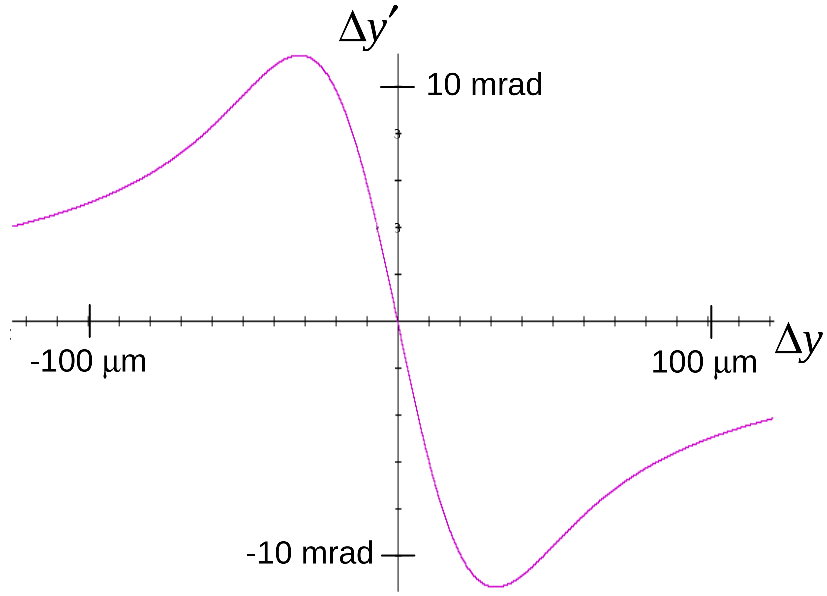
$$|m| + |n| = resonance \;\; order$$

# Nonlinear Beam-Beam

**No analytical solutions for the non-linear beam-beam case.**

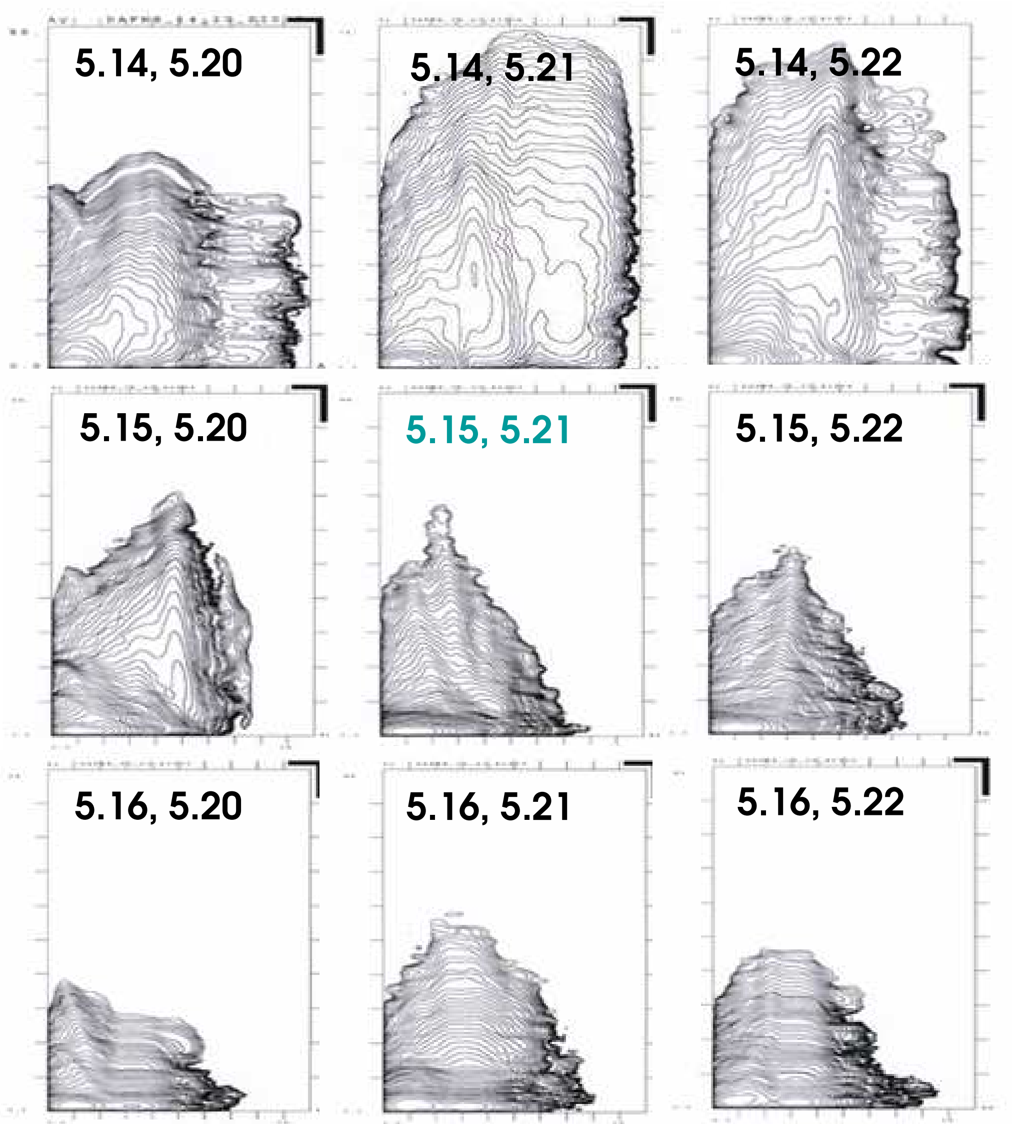
**Simulation Codes**

- **Nonlinear Beam-beam Kick**
- **Synchro-betatron Effects**
- **Radiation Damping**
- **Quantum Fluctuations in Synchrotron Radiation Emission**
- **Lattice Nonlinearities (sextupoles, higher order multipoles)**
- **RF Nonlinearities**

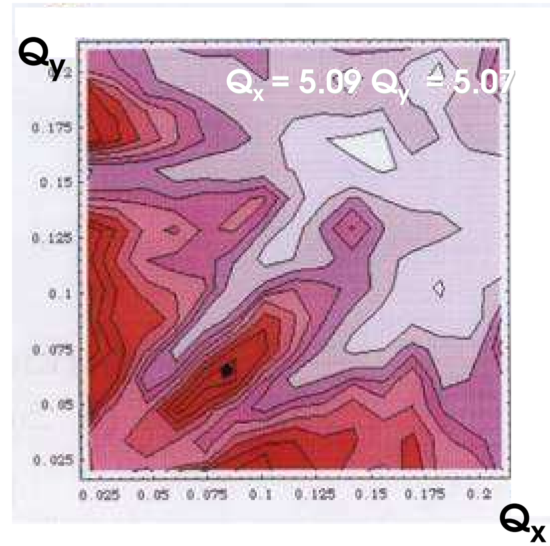


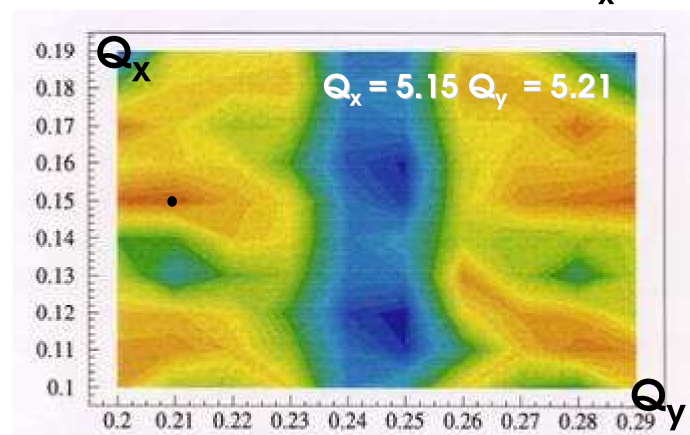

**For example:**
**LIFETRACK by Shatilov**
**or**
**BBC by Hirata**

# Beam-Beam Simulations




**Beam-beam simulations for the DAΦNE collider.**



# Maximum Linear Beam-Beam Tune Shift

**The beam-beam interaction actually sets the maximum achievable luminosity in practically all the existing colliders.**

**No consistent and exhaustive theory exists.**

**The linear beam-beam parameter is used a measure of the strength of the beam-beam interaction.**

**Estimate of the maximum achievable linear tune shift:**

- **Simulations**
- **Statistical analysis of the maximum linear tune shifts achieved in existing colliders**

# Single Bunch Luminosity In Max Linear Tune Shift Regime

$$L = f_R \frac{N_+ N_-}{4\pi \, \sigma_x^* \sigma_y^*}$$

$$\xi_w = \frac{N \, r_e \beta_w^*}{2\pi\gamma \, \sigma_w^* \left(\sigma_y^* + \sigma_x^*\right)} \qquad w = x, y$$

$$N_+ = N_- = N$$

$$\sigma_x = \sqrt{\frac{1}{1+\kappa} \varepsilon \beta_x} \qquad \sigma_y = \sqrt{\frac{\kappa}{1+\kappa} \varepsilon \beta_y}$$

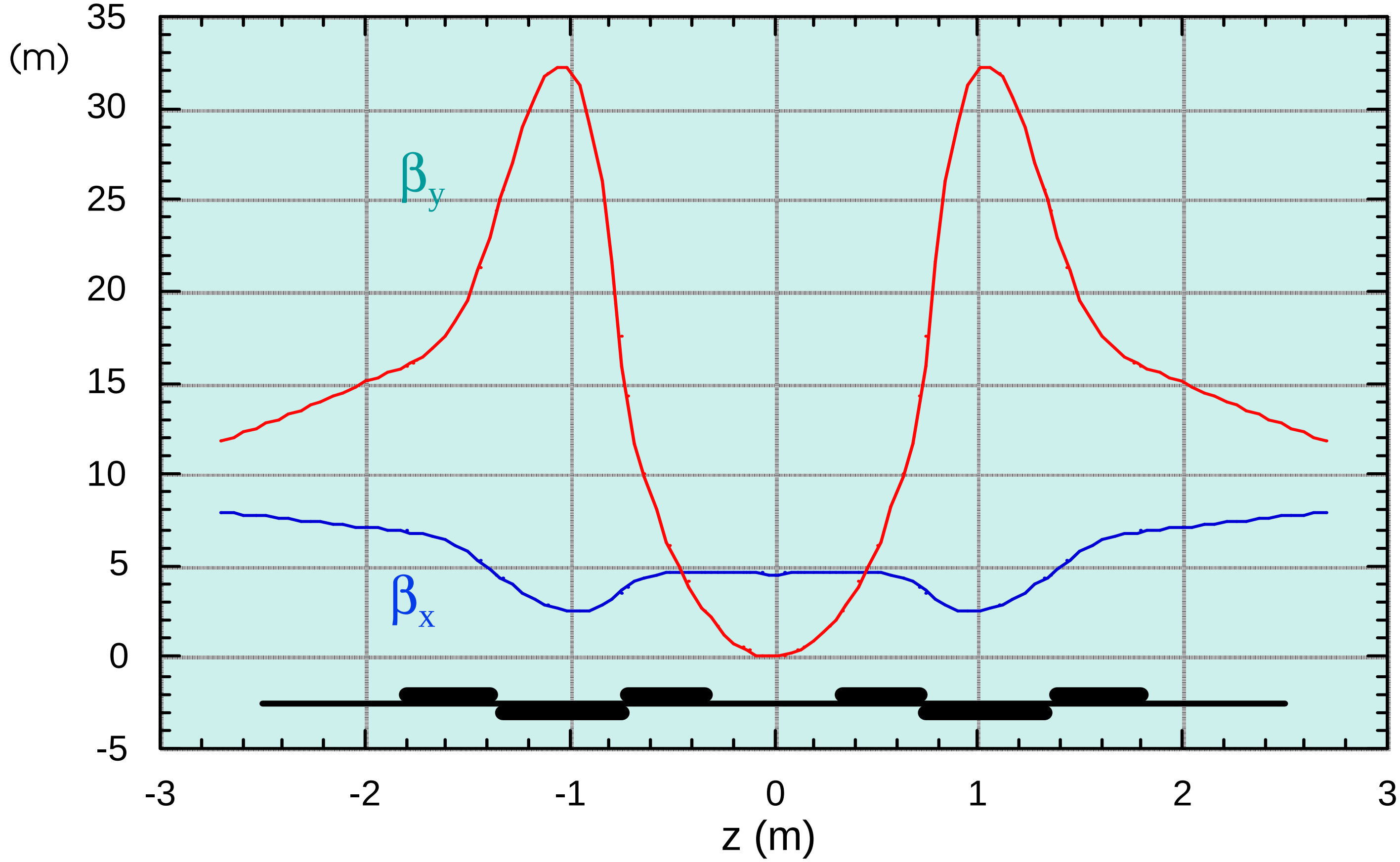
$$\boxed{L = \pi \left(\frac{\gamma}{r_e}\right)^2 f_R \frac{\varepsilon}{1+\kappa} \left(\frac{1}{\sqrt{\beta_y^*}} + \sqrt{\frac{\kappa}{\beta_x^*}}\right)^2 \xi_x \, \xi_y}$$

**Equal tune shift design**

$$\frac{\beta_y^*}{\beta_x^*} = \kappa \quad \Leftrightarrow \quad \xi_x = \xi_y = \xi$$

$$\boxed{L = \pi \left(\frac{\gamma}{r_e}\right)^2 f_R \, \varepsilon (1+\kappa) \frac{\xi^2}{\beta_y^*}}$$

# Low Beta Scheme

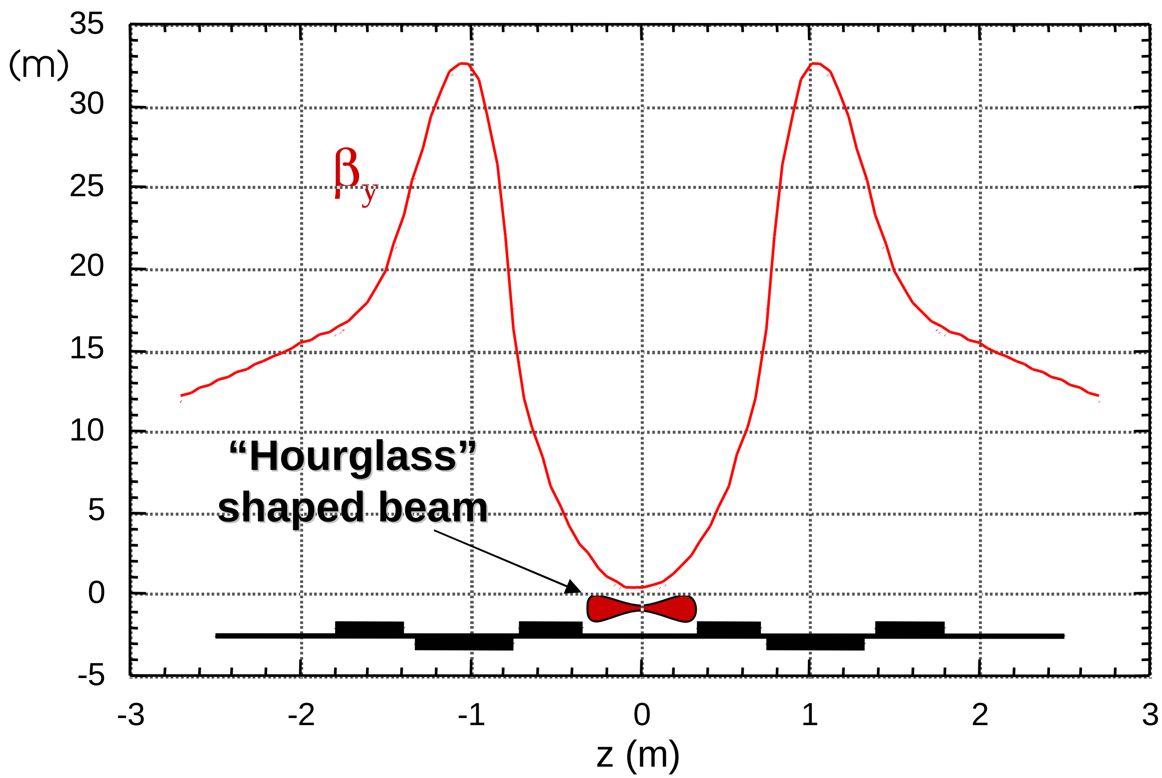



**Few centimeters vertical beta @ IP are "routinely" obtained.**

# Hourglass Effect

**In a low beta insertion, the beta function between the IP and the first quadrupole is given by:**

$$\beta_y(z) = \beta_y^*\left[1+\left(\frac{z}{\beta_y^*}\right)^2\right] \Rightarrow \sigma_y^2 = \varepsilon_x\beta_y(z) = \varepsilon_x\beta_y^*\left[1+\left(\frac{z}{\beta_y^*}\right)^2\right]$$




**The hourglass effect can limit the luminosity achievable in low beta schemes.**

$$L = \pi\left(\frac{\gamma}{r_e}\right)^2 f_R\,\varepsilon(1+\kappa)\,\frac{\xi^2}{\beta_y^*}$$

**The use of short bunches can reduce the effect on luminosity.**

# Low Beta Scheme Implications

## Large Vertical Beta Functions in D Quads @ IR

Larger Negative Values of Vertical Chromaticities
Stronger Correcting Sextupoles
Smaller Dynamic Aperture
Decrease of Beam Lifetime

## Short Bunches for Minimizing the Hourglass Effect

Increase of Touschek Effect
Decrease of Beam Lifetime
Higher Frequency Components in the Beam Spectrum
Possible Coupling with High Frequency
Vacuum Chamber Modes: Instabilities
Higher Peak RF Voltages: Larger Number of Cavities
RF Nonlinearities, Stronger High Order Modes
Coherent Synchrotron Radiation
with High Current per Bunch

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# Round Beam vs Flat Beam

## Round Beam (k~1):

A Factor 2 of Luminosity Gain
Both the Beta Functions @ IP Must Be Small:
Technically Difficult to Obtain
Large Negative Chromaticities in Both Planes
Strong Sextupole Correction
Small Dynamic Aperture
Strong Beam-beam Effects

## Flat Beam (k<<1):

A Factor 2 of Luminosity Loss
Chromaticity Handling not Critical:
It is Possible to Arrange the
Collider Parameters in Order to
Obtain Better Luminosity Performances

# Multibunch Luminosity

$$L = N_B L_{SB}$$

## Large Number of Bunches:

- Separate Rings
- Small Distance Between 2 adiacent bunches
- Multibunch Instabilities
- Low Impedance Vacuum Chamber
- HOM 'Free' Ring Components
- Longitudinal Feedback System
- Horizontal Crossing Angle @ IP Required in Order to Avoid Parasitic Crossing
- Synchro-betatron Resonances

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- Larger Stored Current
- Vacuum System Limitations
- Large RF Power
- Vacuum Chamber Large Heating Load

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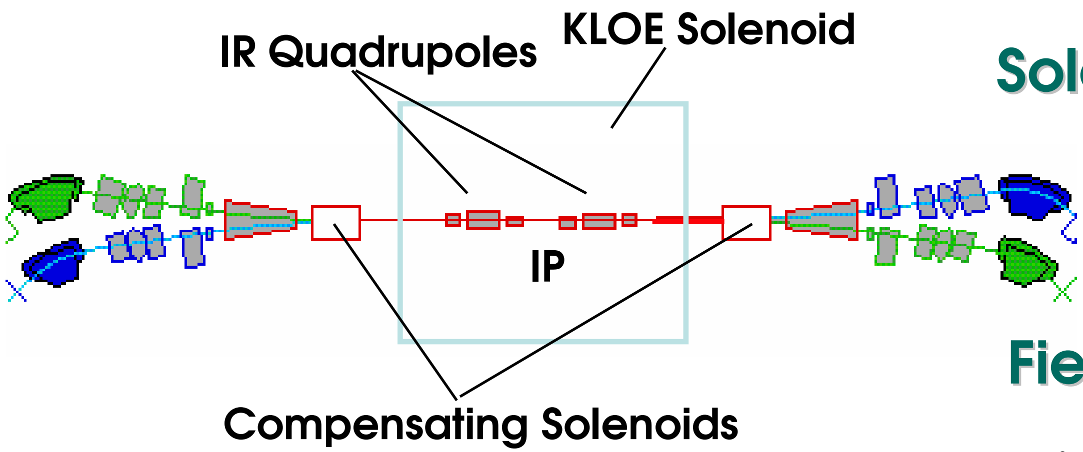
# Detector Implications

The Solenoidal Field Introduces Coupling between the Vertical and Horizontal Planes that Must Be Carefully Corrected.

Experimental Requirements Concerning Solid Angle Stay Clear Forced to Have Permanent IR Quadrupoles and a Very Reduced Configuration of Beam Dignostics

# Detector Effects Compensation: DAΦNE example




## Solenoid Frame Rotation Angle:

$$\theta_S = \frac{1}{2(B\rho)}\int_{z_1}^{z_2} B_z(s)\,ds$$

## Field Integral Compensation:

$$\int_{Comp.1} B_z(s)\,ds + \int_{KLOE} B_z(s)\,ds + \int_{Comp.2} B_z(s)\,ds = 0$$

## Rotated IR Quadrupoles to correct Coupling:

$$\theta_n^Q = \frac{1}{2(B\rho)}\int_{IP}^{C_n} B_z(s)\,ds \quad n = 1,2,3 \qquad C_n \equiv \text{n} - \text{th quad center position}$$

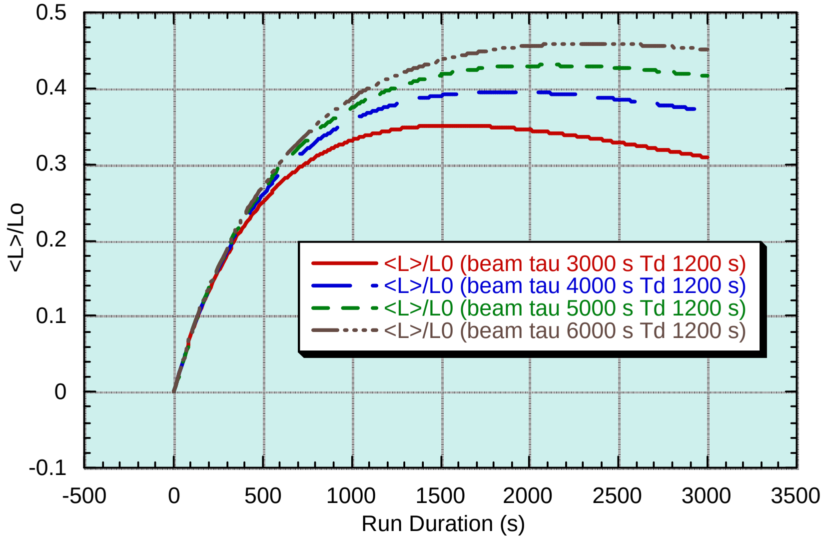
$$B_z = 0.6\ T \qquad (B\rho) = 1.70\ T\ m$$

$$C_1 = 0.53\ m \qquad C_2 = 1.04\ m \qquad C_3 = 1.59\ m$$

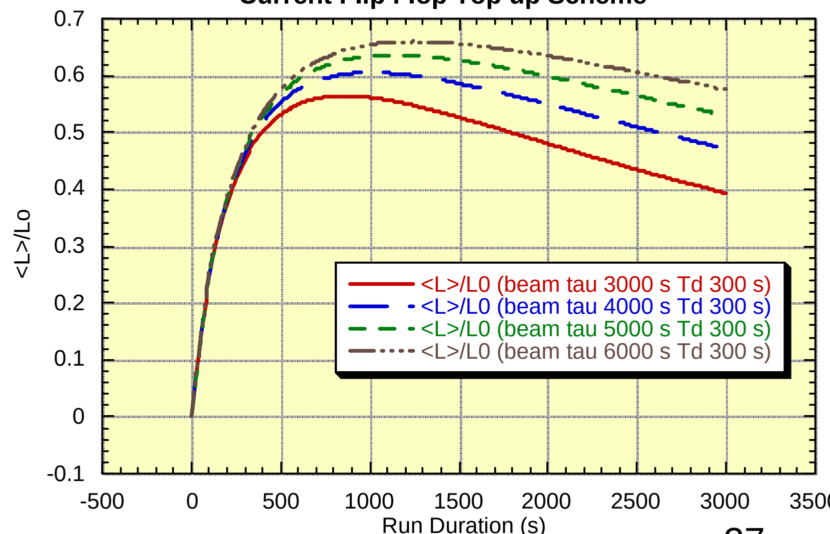
$$\boxed{\theta_1^Q = 5.35\ \text{deg} \quad \theta_2^Q = 10.5\ \text{deg} \quad \theta_3^Q = 16.1\ \text{deg}}$$

# Integrated Luminosity Optimization

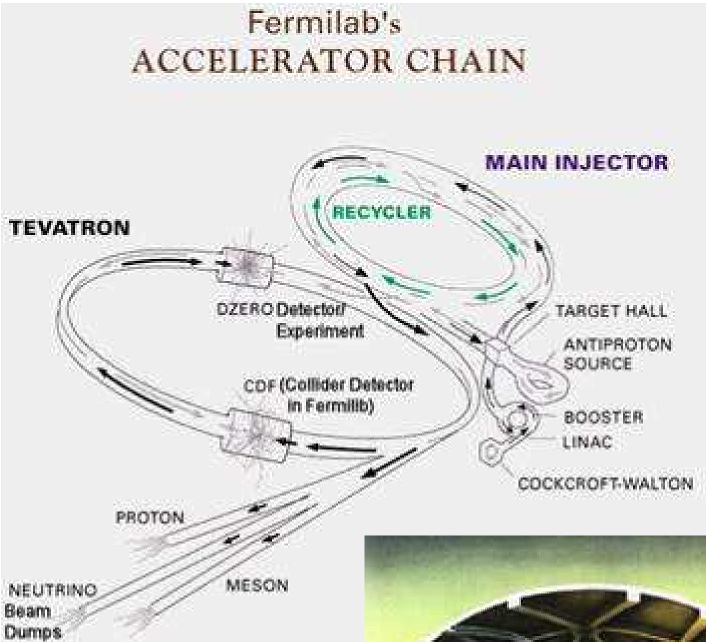
**Average Luminosity vs Run Duration**
**Complete Injection Scheme**

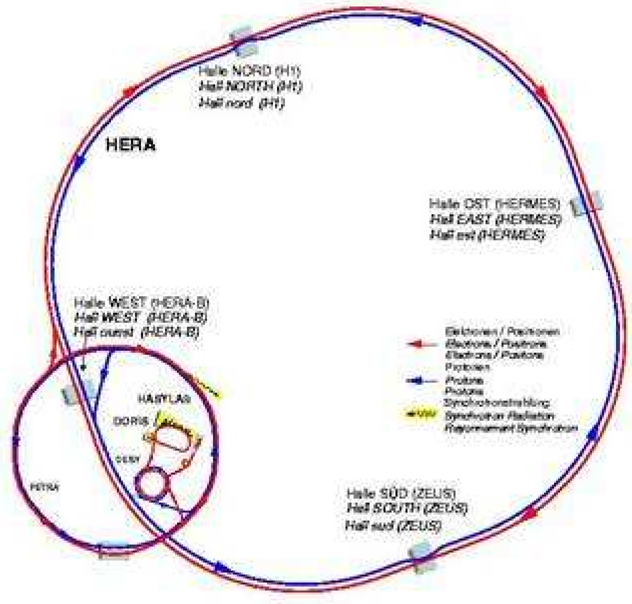


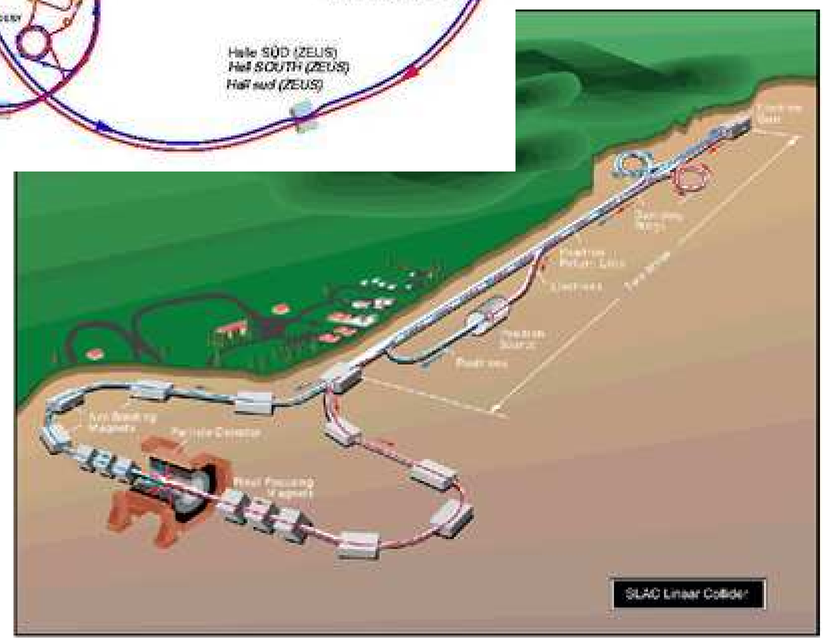

**Average Luminosity vs Run Duration**
**Current Flip Flop Top up Scheme**



# Examples of Colliders



# Examples of Colliders

**KEK
B-Factory**